UNCERTAINTY IN CONTACT ANGLE ESTIMATES FROM THE TANGENT METHOD

INTRODUCTION
When measuring contact angles ($\theta$) by the tangent method, how much uncertainty is there?

OBJECTIVE
Determine the uncertainty in measurements of $\theta$ from the tangent method across the full range of wettability, from $\theta = 0^\circ$ to $\theta = 180^\circ$.

ANALYSIS
Contact angle ($\theta$) from slopes of the baseline ($m_b$) and tangent line ($m_t$)

$$\theta = \text{ArcTan} \left( \frac{m_t - m_b}{1 + m_t \cdot m_b} \right)$$

Estimate uncertainty in contact angles ($\delta \theta$) using standard error propagation techniques,

$$\delta \theta = \left[ \left( \frac{\partial \theta}{\partial m_t} \right)^2 (\delta m_t)^2 + \left( \frac{\partial \theta}{\partial m_b} \right)^2 (\delta m_b)^2 \right]^{1/2}$$

where absolute uncertainties are $\delta m_b$ and $\delta m_t$, and corresponding relative uncertainties are

$$\Delta_i = \frac{\delta m_i}{m_i}, \quad \Delta_b = \frac{\delta m_b}{m_b}$$

Insert partial derivatives and rearrange terms

$$\delta \theta = \frac{1}{1 + m_t^2} \left( (\delta m_t)^2 + (\delta m_b)^2 (1 + m_t^2)^2 \right)^{1/2}$$

Contact angle ($\theta$) is related directly to the slope of the tangent line ($m_t$) through the tangent function,

$$|m_t| = \text{Tan} \theta$$

Further simplify relative uncertainties

$$\Delta = \Delta_i = \Delta_b = \frac{\delta m_t}{m_t}$$

Recast $\delta \theta$ as

$$\delta \theta = \text{Tan} \theta \left[ 1 + \cos^4 \theta \right]^{1/2} \Delta$$

RESULTS

CONCLUSION
Uncertainty in $\theta$ from the tangent method:
Small for low to moderate $\theta$ values
Increases asymptotically near $\theta = 90^\circ$