Repellency of the Lotus Leaf: Contact Angles, Drop Retention and Sliding Angles
Chuck Extrand and Sung In Moon
June 2015
OUTLINE

• Introduction
• Objective
• Analysis
• Comparison to Experiments
• Conclusions

Source: wikipedia.org/wiki/Nelumbo_nucifera

Source: india-forums.com
INTRODUCTION

INTRODUCTION
INTRODUCTION
INTRODUCTION
INTRODUCTION
INTRODUCTION
INTRODUCTION
INTRODUCTION
INTRODUCTION

Langmuir

Repellency of the Lotus Leaf: Resistance to Water Intrusion under Hydrostatic Pressure

C. W. Extrand

Entegris, Inc., 101 Peavey Road, Chaska, Minnesota 55318, United States

**ABSTRACT:** In an attempt to better understand the repellency of the lotus leaf, a model was constructed from hydrophobic hemispheres arranged on a hexagonal array. Two scenarios were considered. In the first, the hemispheres were smooth. In the second, the hemispheres had a secondary roughness. The model shows that, without the secondary structure, the repellency of this surface geometry is relatively poor. The secondary structure directs the surface tension upward, allowing much greater resistance to penetration of water and prevents the loss of repellency. From the proposed model, the maximum intrusion pressure (or so-called Cassie—Wenzel transition) of the lotus leaf is estimated to be 12—15 kPa. The predicted maximum pressure agrees well with reported values from experimental measurements.
OBJECTIVE

• Create lotus leaf model
• Estimate:
  - Intrusion
  - Contact
  - Contact angles
  - Sliding angles
• Compare to experimental observations

Image kindly provided by Prof. W. Barthlott
ANALYSIS

Image kindly provided by Prof. W. Barthlott

\[ 2R = 11.0 \pm 1.4 \, \mu m \]
\[ 2y = 18.6 \pm 3.3 \, \mu m \]
ANALYSIS
ANALYSIS

• Advancing Edge (and Contact Patch)

- $\phi \equiv$ intrusion angle
- $\theta_{a,0} \equiv$ intrusion angle
- $\gamma \equiv$ liquid surface tension
- $R \equiv$ protuberance radius
ANALYSIS

• Drop Dimensions and Volumes

- $\theta \equiv$ apparent contact angle
- $V \equiv$ drop volume
- $b \equiv$ apex radius
- $h \equiv$ drop height
- $a \equiv$ contact radius

Bashforth & Adams 1883
### ANALYSIS

- Water Drop Volumes ($V$) and Dimensions ($b$, $2a$, $h$)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$V$ (µL)</th>
<th>$b$ (mm)</th>
<th>$2a$ (mm)</th>
<th>$h$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>3.4</td>
<td>0.959</td>
<td>0.496</td>
<td>1.71</td>
</tr>
<tr>
<td>0.25</td>
<td>8.7</td>
<td>1.36</td>
<td>0.907</td>
<td>2.24</td>
</tr>
<tr>
<td>0.50</td>
<td>21</td>
<td>1.92</td>
<td>1.57</td>
<td>2.83</td>
</tr>
<tr>
<td>0.75</td>
<td>33</td>
<td>2.35</td>
<td>2.10</td>
<td>3.18</td>
</tr>
</tbody>
</table>

Bashforth & Adams 1883
ANALYSIS

• Intrusion Angle ($\phi$)

$$\Delta p_c - \Delta p_h - \Delta p_L = 0$$

$$\frac{2R \gamma \sin \phi \cos(\theta_{a,0} + \phi)}{(2\sqrt{3}/\pi)y^2 - R^2 \sin^2 \phi} + \rho gh + \frac{2\gamma}{b} = 0$$

- $\Delta p_c \equiv$ capillary pressure
- $\Delta p_h \equiv$ hydrostatic pressure
- $\Delta p_L \equiv$ Laplace pressure
ANALYSIS

- Intrusion Angle ($\phi$)

\[
\Delta p_c - \Delta p_h - \Delta p_L = 0
\]

\[
\frac{2R\gamma \sin \phi \cos (\theta_{a,0} + \phi)}{(2\sqrt{3}/\pi)y^2 - R^2 \sin^2 \phi} + \rho gh + \frac{2\gamma}{b} = 0
\]

\[
\phi = 2.2 - 3.6^\circ
\]

- $\Delta p_c \equiv$ capillary pressure
- $\Delta p_c \equiv$ hydrostatic pressure
- $\Delta p_c \equiv$ Laplace pressure
ANALYSIS

• Linear Fraction of Contact ($\lambda_p$) with Protuberances

$$\lambda_p = \frac{l_p}{l_p + l_b} = \frac{1}{1 + \left(\frac{2}{\pi}\right)\left(\frac{y}{R} - 1\right) \csc \phi}$$
ANALYSIS

• Linear Fraction of Contact ($\lambda_p$) with Protuberances

\[ \lambda_p = \frac{l_p}{l_p + l_b} = \frac{1}{1 + \left(\frac{2}{\pi}\right)\left(\frac{y}{R} - 1\right) \csc \phi} \]

\[ \lambda_p = 0.08 - 0.13 \]
ANALYSIS

- Apparent Contact Angles ($\theta_i$)

$$\theta_i = \lambda_p \theta_{p,i} + \lambda_b \theta_b$$

ANALYSIS

- Apparent Advancing Angles ($\theta_a$)

$$\theta_a = \lambda_p (\theta_{a,0} + \phi + 90^\circ) + \lambda_b \cdot 180^\circ$$
ANALYSIS

- Apparent Advancing Angles ($\theta_a$)

\[
\theta_a = \lambda_p (\theta_{a,0} + \phi + 90^\circ) + \lambda_b \cdot 180^\circ
\]

$\theta_a \equiv 180^\circ$
COMPARISON TO EXPERIMENTS

• Apparent Advancing Contact Angles

3.4 µL  8.7 µL

21 µL  33 µL
COMPARISON TO EXPERIMENTS

• Apparent Advancing Contact Angles

\[ \theta_a = 180^\circ \]

<table>
<thead>
<tr>
<th>Volume (µL)</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td><img src="3.4%C2%B5L.png" alt="Image" /></td>
</tr>
<tr>
<td>8.7</td>
<td><img src="8.7%C2%B5L.png" alt="Image" /></td>
</tr>
<tr>
<td>21</td>
<td><img src="21%C2%B5L.png" alt="Image" /></td>
</tr>
<tr>
<td>33</td>
<td><img src="33%C2%B5L.png" alt="Image" /></td>
</tr>
</tbody>
</table>
ANALYSIS

- Apparent Receding Angles ($\theta_r$)

Choi et al., JCIS 2009

Dufour et al., Small 2013
ANALYSIS

- Apparent Receding Angles ($\theta_r$)

$$\theta_r = \lambda_p (\theta_{r,0} + \phi - 90^\circ) + \lambda_b \cdot 180^\circ$$
ANALYSIS

- Apparent Receding Angles ($\theta_r$)

\[ \theta_r = \lambda_p (\theta_{r,0} + \phi - 90^\circ) + \lambda_b \cdot 180^\circ \]

\[ \theta_r = 158 - 160^\circ \]
COMPARISON TO EXPERIMENTS

• Apparent Receding Contact Angles

<table>
<thead>
<tr>
<th>Volume</th>
<th>Appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 μL</td>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>8.7 μL</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>21 μL</td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>33 μL</td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>
COMPARISON TO EXPERIMENTS

• Apparent Receding Contact Angles

\[
3.4 \, \mu L \quad 8.7 \, \mu L \quad 21 \, \mu L \quad 33 \, \mu L
\]

160° ≤ θ_r < 180°
ANALYSIS

- Sliding Angles ($\alpha_a$) from Contact Angles

$$\sin \alpha_a = k \frac{\gamma \cdot a}{\rho g V} (\cos \theta_r - \cos \theta_a)$$
ANALYSIS

- Sliding Angles ($\alpha_a$) from Contact Angles

$$\sin \alpha_a = k \frac{\gamma \cdot a}{\rho g V} (\cos \theta_r - \cos \theta_a)$$

$$\alpha_a = 0.6 - 3.3^\circ$$
ANALYSIS

- Sliding Angles ($\alpha_b$) from Capillary Bridge Rupture

$$\sin \alpha_b = \frac{\pi^2 \gamma \cdot aR}{\rho g V_y} \sin \phi$$
ANALYSIS

- Sliding Angles ($\alpha_b$) from Capillary Bridge Rupture

\[
\sin \alpha_b = \frac{\pi^2 \gamma \cdot aR}{\rho g V_y} \sin \phi
\]

\[
\alpha_b = 3.0 - 11^\circ
\]
COMPARISON TO EXPERIMENTS

• Sliding Angles
COMPARISON TO EXPERIMENTS

- Sliding Angles

\[ \alpha < 5-10^\circ \]
COMPARISON TO EXPERIMENTS

• Sliding Angles

\[ \alpha_a < 0.6 - 3.0^\circ \quad \alpha_b < 3.0 - 11^\circ \]

[Diagram showing angles and their respective ranges]
CONCLUSIONS

• Wetting behavior of the lotus leaf depends on drop size and shape
• Smaller drops penetrate deeper
  - Advancing angles: 180°
  - Receding angles: between 160° and 180°
  - Sliding angles: ≤ 10°
QUESTIONS AND COMMENTS?

Source: http://en.wikipedia.org/wiki/Nelumbo_nucifera

Source: http://en.wikipedia.org/wiki/Nelumbo_nucifera